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# SUPERRESOLVED IMAGING USING THE HARTLEY TRANSFORM AND A HEMISPHERICAL OBJECT SPACE

THESIS

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THESIS

Presented to the Faculty of the School of Engineering
of the Air Force Institute of Technology
Air University
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Master of Science in Engineering Physics

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## Preface

One limit to the information retrievable from an image is the "diffraction limit" which translates to limited resolution. Superresolved imaging is a field devoted to increasing resolution beyond this limit. Whereas iterative superresolution has been shown to work in general, it traditionally requires a knowledge of the finite dimensions of the object. This thesis demonstrates that the same technique can be used without this knowledge. The technique has also been modified to use the more efficient Hartley transform instead of the Fourier transform.

As indirect contributors to the results shown herein, my thanks go to the following: my wife Sandra, for being her; Dr. Theodore E. Luke, for the long leash; Dr. Steven K. Rogers for his advice on phase retrieval; LtCol James P. Mills for challenging my work; Bruce Hornsby, Pink Floyd and Rush for inspiration; and John T. Kelly, my father, for always asking me "Why?".

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## ABSTRACT

Resolution in an image can be increased by an iterative technique (introduced by Gerchberg) which effectively continues the known, partial spectrum beyond the limits imposed by an optical system. This increased resolution is called superresolution. Historically, an important constraint assumed for this technique was the knowledge of the finite dimensions of the object such that the object energy outside these dimensions must be zero. It is shown in this thesis that by a change of object space geometry, the semicircular field of view of an optical system provides a natural dimensional constraint which can be used instead of the object dimensions to achieve superresolution. A further modification of the iterative technique involves using the Fast Hartley Transform (FHT) instead of the Fast Fourier Transform (FFT). The FHT is inherently faster and requires less computer memory than the FFT.

## SUPERRESOLVED IMAGING USING THE HARTLEY TRANSFORM AND A SPHERICAL OBJECT SPACE

### I. INTRODUCTION

According to Gerchberg (1:711) and supported by Fienup (2:161), seemingly lost spatial information about an object might be recoverable if constraints are known in both the object and object spatial frequency domains. The transform generally relating these domains is the Fourier transform. One constraint in object space which is commonly used is the finite dimension of the object. However, if this knowledge is gained from received data (ie autocorrelation data), then it may be highly suspect in real scenarios where detector noise, diffracted light and undesireable objects in the field of view retard the desired object's autocorrelation properties.

The recovery of lost spatial information is of great importance to both civilian and military applications of imaging. One loss mechanism is derived from the relationship between the extent of the received spatial frequency function (the spectrum) and the maximum resolution obtainable from a given optical system aperture. As the aperture size increases, higher resolution is possible due to the larger spectrum passing through the aperture. However, there are

many practical restrictions to aperture size. Another method of increasing resolution is to continue the finite or "band-limited" spectrum of a small aperture further into frequency space to simulate using a larger aperture. This is possible due to the analytic nature of an object's spectrum (3:932). Such a continuation can be accomplished by the Gerchberg technique given sufficient constraints. The enhanced spectrum can then be transformed into an image with higher resolution than the image formed with the initial spectrum. Other investigations of superresolved imaging can be found in appendix A.

The purpose of this thesis is to examine an alternative constraint in object space and apply this constraint to the iterative superresolution technique. The new constraint naturally appears when a spherical object space is chosen instead of the traditional infinite rectangular object space. Also, since this technique is iterative, a fast Hartley transform will be used instead of the more obvious fast Fourier transform due to the former's greater speed and smaller memory requirements. The development will only consider a one-dimensional object (ie an object function of only one variable).

## II. BACKGROUND

### Iterative Superresolution.

To paraphrase Gerchberg (1:712), the superresolution algorithm is

as follows. The band-limited spectrum is inverse-Fourier transformed to create an image. The energy which falls outside of the known object dimensions in image space is set to zero. This modified image function is Fourier transformed to generate a new spectrum. The new spectrum is replaced by the initial band-limited spectrum only within that band-limitation. Repeating this routine gradually builds the initial spectrum out beyond the limitations imposed by the optical system which generated it.

The explanation of the above method's ability to continue the finite spectrum is fairly simple. The actual spectrum of any finite object continues in frequency space indefinitely. Thus, the band-limited spectrum can be considered as the actual spectrum plus an "error" spectrum which is first, equal and opposite the actual spectrum outside the frequency band passed by the optical system and second, zero inside this band (making the total spectrum outside the band zero). The linear properties of the Fourier transform allow separate treatment of each constituent spectrum. The actual spectrum obviously inverse-transforms to the original object function. The error spectrum, being zero for a finite region and generally non-zero elsewhere, will inverse-transform to an image error function which has energy outside the object constraints in image space. Since this is the energy which is zeroed in the algorithm, then by Parseval's theorem, only the energy of the error spectrum is decreased. As the error spectrum decreases, the actual spectrum is revealed.

## Use of the Hartley Transform.

As an important note, the above superresolution technique is not necessarily confined to a Fourier transform space relation. Such a scenario is commonly found in optics however, and justifies its initial establishment. In general, any transform/inverse-transform relation which maps energy beyond constraints in one space and has a constraint in the other space should work. While the Hartley transform provides one of these relations, it will not be introduced as such. A Fast Hartley Transform (FHT) will be used in this thesis only in its capacity to make the superresolution algorithm more efficient in computation time and computer memory requirements. Bracewell (4:12) has shown that the Hartley transform, h(x), can be determined from the Fourier transform, f(x), by

$$h(x) = f_{real}(x) - f_{imag}(x) \tag{1}$$

Since the ultimate objective is a superresolved image, not necessarily a continued Fourier spectrum, conversion back to the Fourier domain is unnecessary. Thus the superresolution algorithm will be performed from image space to frequency space via the FHT, with the frequency function called the Hartley spectrum. The algorithm is summarized in Figure one. The convergence criterion is simply a given number of iterations.

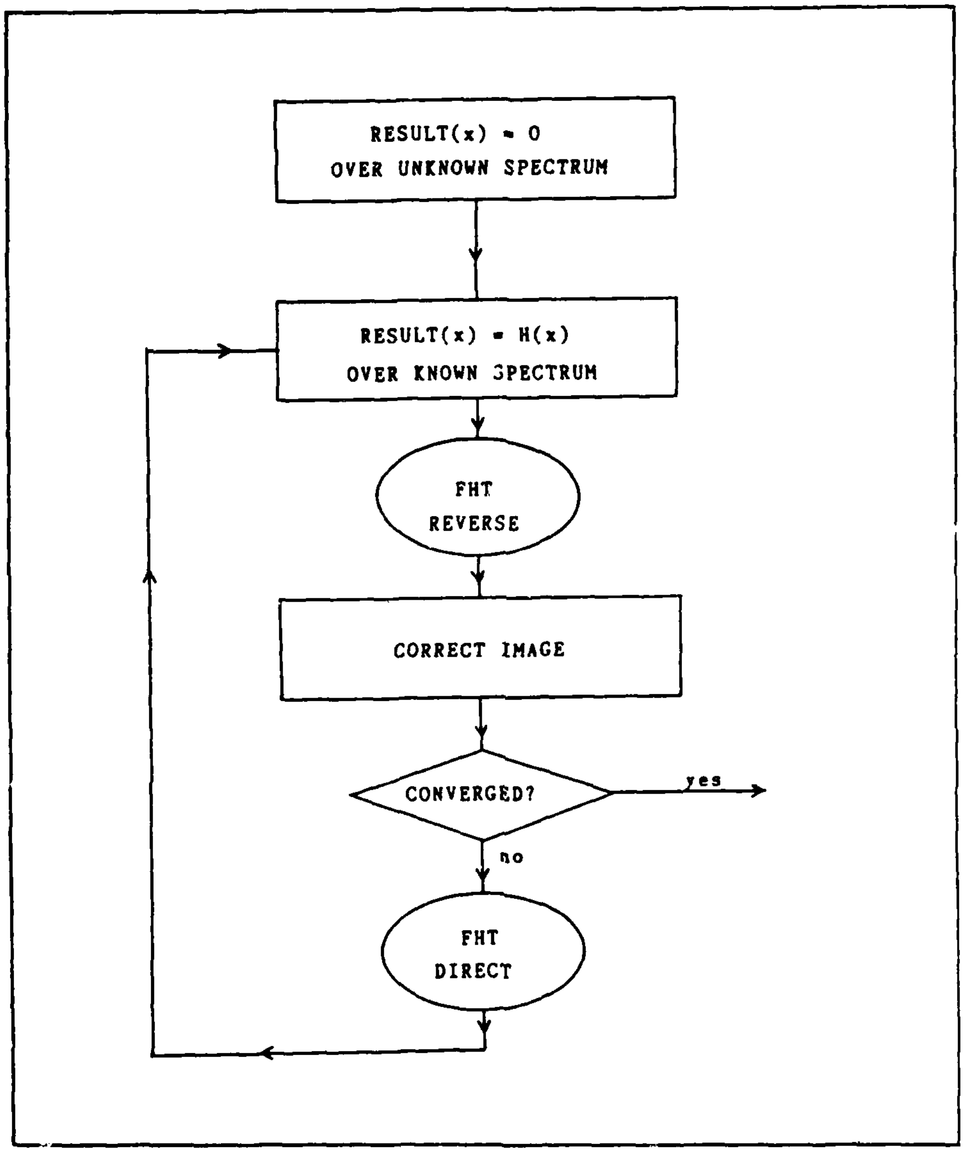


Figure 1. Superresolution Flow Diagram.

The constraint in frequency space is now the known, band-limited Hartley spectrum converted from the Fourier spectrum. The array "RESULT" contains the discrete values of either the Hartley spectrum or image functions, depending on the step of the algorithm.

## III. THEORY

The relation between the two spaces used in the superresolution algorithm is defined empirically by the propagation of spherical waves from the object in accordance with the Fresnel-Kirchhoff diffraction formula which assumes that the separation between spaces is much greater than the wavelength of radiation observed (5:41). If the object and optical system aperture dimensions are also much smaller than this separation, then it is common practice to approximate the diffraction formula, and thus the space relation, with a Fourier transform. It is important to emphasize that this relation only applies to a small maximum object dimension. Even though the rectangular object space historically used extends indefinitely, the results of an inverse-Fourier transform which map outside the small region can not be assumed accurate because such an operation violates the geometry.

There are two reasons for choosing a spherical object space over the rectangular object space. First, the propagation geometry from such a curved space should more closely approximate a Fourier transform over the entire object space. Second, with a spherical space, only a hemisphere of light can enter a windowed aperture plane. This places a natural constraint on the object space which may be used to reduced error energy.

Transform Formulation.

The traditional diffraction formula can be written as

$$D(x) = c\int_{-\pi/2}^{\pi/2} O(\alpha)\frac{\exp(ikr)}{r} Z(x,\alpha,r)d\alpha \qquad (2)$$

where D(x) is a complex distribution at the system aperture plane, c is a complex constant, $O(\alpha)$ is the object function, $k = 2\pi/\lambda$ ($\lambda$ is the wavelength) and $Z(x,\alpha,r)$ is the obliquity function (5:58). The geometry is shown in Figure 2.

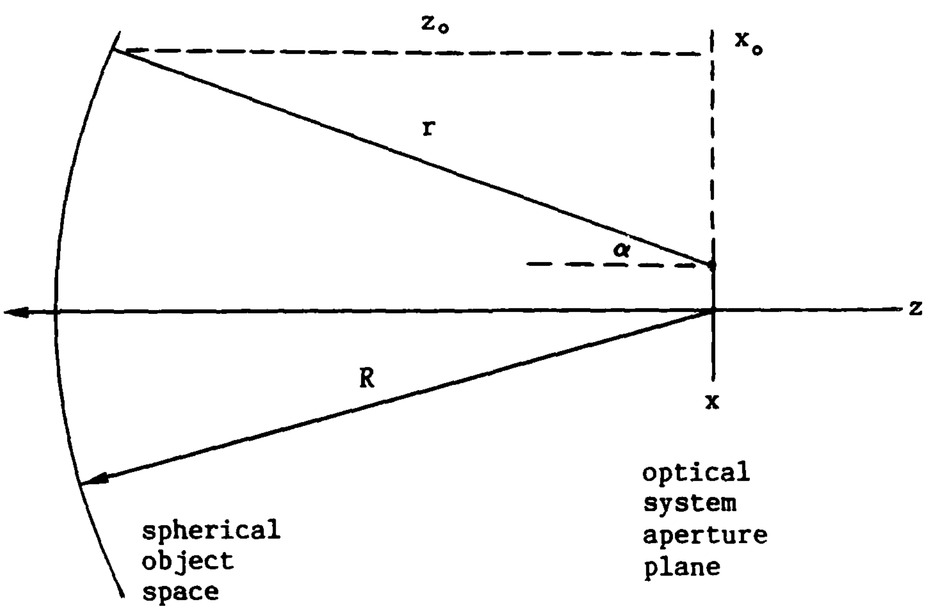


Figure 2. Propagation from a Spherical Object Space.

The limits of integration are representative of the semicircular geometry. Of course there may be object energy outside this semicircle, but none of the energy which enters the system aperture is from this outside regime. We make an initial but standard approximation that if R is very large, then $r \approx R$ for all <u>amplitude</u> terms. This also affects the obliquity function such that it becomes only a function of $\alpha$. Thus

$$D(x) = \frac{c}{R}\int_{-\pi/2}^{\pi/2} O(\alpha)\exp[ikr]Z(\alpha)d\alpha \qquad (3)$$

We now explore the <u>phase</u> term. The distance $r^2$ can be written as $(x_o-x)^2+(z_o)^2$ by the pythagorean theorem. In spherical coordinates, $z_o = R\cos(\alpha)$ and $x_o = R\sin(\alpha)$ . Thus

$$r = \sqrt{R^2\sin^2(\alpha) - 2xR\sin(\alpha) + x^2 + R^2\cos^2(\alpha)} \qquad (4)$$

or

$$r = R\sqrt{1 + [x^2/R^2 - 2(x/R)\sin(\alpha)]} \qquad (5)$$

We rename the expression in brackets b. In examining the magnitude of b, with the scenario that $x<<R$, we find two extreme cases depending on the magnitude of the sine function. If $\sin(\alpha) = 1$ or $-1$ then the magnitude of b is given by $(x/R)(x/R - 2)$, whereas if $\sin(\alpha) = 0$ then this magnitude becomes $(x/R)^2$. In both cases, the magnitude of

b is less than 1. A binomial series expansion can therefore be used which is formally

$$[1+b]^{1/2} = 1 + \frac{1}{2}b - \frac{1}{(2)(4)}b^2 + \frac{(1)(3)}{(2)(4)(6)}b^3 \quad . \quad . \quad . \qquad (6)$$

The $\sin(\alpha)$ being identified as P, the expansion is applied to yield separate terms for r from eqn (5). After some algebra,

$$r = R - xP + \frac{x^2}{R}[1/2 + P^2] + \frac{x^3P}{2R^2} - \frac{x^4}{8R^3} + Q \qquad (7)$$

where Q represents terms with ratios of $x^{n+1}/R^n$ with n being 2 or above. Each term is then examined to appraise its impact on the result of exp[ikr] when substituted. We choose for example a satellite at 100 km viewed from a 10 cm optical system with a wavelength at $5\times10^{-7}$ meters. The maximum values for the variables are applied for each term.

1. R is a constant and thus comes out of the integral.
2. kxP varies from + to − $(2\pi/5\times10^{-7})(5\times10^{-2}) = 6.28\times10^{6}$.
3. Term 3 varies from 0 to $(2\pi/5\times10^{-7})(5\times10^{-2})^2/(1\times10^{5})$, a difference of $\pi/10$ or 5% of a phase angle.
4. Term 4 varies from 0 to $(\pi/5\times10^{-7})(5\times10^{-2})^3/(1\times10^{10})$, a difference of $7.85\times10^{-8}$, which is negligible.
5. Terms 5 and 6 involve ratios of $x^{n+1}/R^n$ smaller than that of term four. Their affect is therefore also negligible.

If we accept the scenario and assume that the 5% variation of

term 3 makes an insignificant contribution to error, then all but the first two terms can be dropped. Thus

$$r = R - x\sin(\alpha) \tag{8}$$

Substituting the approximation for r back into eqn (3) and bringing the constant term out of the integral to form a new complex constant $c_1$ yields

$$D(x) = c_1 \int_{-\pi/2}^{\pi/2} O(\alpha)Z(\alpha)\exp[-ikx\sin(\alpha)]d\alpha \tag{9}$$

If we let $q = \sin(\alpha)/\lambda$ , then $dq/d\alpha = \cos(\alpha)/\lambda$ . The limits thus change to form the expression (with a new aggregate constant $c_2$)

$$D(x) = c_2 \int_{-1/\lambda}^{1/\lambda} O(\arcsin(q\lambda))Z(\arcsin(q\lambda))\cos^{-1}(\arcsin(q\lambda))\exp[-2\pi ixq]dq \tag{10}$$

If we define $S(\arcsin(q\lambda)) = c_2 Z(\alpha)\cos^{-1}(\alpha)$ , then

$$D(x) = \int_{-1/\lambda}^{1/\lambda} O(\arcsin(q\lambda))S(\arcsin(q\lambda))\exp[-2\pi ixq]dq \tag{11}$$

Finally, if the object and modified obliquity functions are transformed by a "q transform" into functions in "q" space, such that $O_q(q) = O(\arcsin(q\lambda))$ and $S_q(q) = S(\arcsin(q\lambda))$ then

$$D(x) = \int_{-1/\lambda}^{1/\lambda} O_q(q)S_q(q)\exp[-2\pi ixq]dq \tag{12}$$

which of course is simply the Fourier transform of the q transformed object function modulated in amplitude by a modified and q transformed obliquity function. ***Whereas the rectangular object space provides Fourier transform properties only in a small object space region, the spherical object space results in approximate Fourier propagation <u>from any object location</u>.*** Also the rectangular space could not accomodate energy below the plane of the optical system aperture since this plane corresponds to infinity in that object space. The spherical space of course continues past this plane.

<u>Field of View versus Obliquity.</u>

It is quite obvious that if the product $O_q(q)S_q(q)$ is recovered, then a simple division by $S_q(q)$ will yield our goal. However, if the original obliquity function $Z(\alpha)$ behaves as a result of vector addition and/or polarization phenomena, then it will likely have something like a $\cos(\alpha)$ dependence, at least near the optical axis. Since the modified obliquity function $S_q(q)$ is a result of $Z(\alpha)\cos^{-1}(\alpha)$, the assumed behaviour of $Z(\alpha)$ will cause a cancellation of the cosine functions. Thus for regions close to the optical axis (for example, a ten degree FOV (field of view)), the obliquity function is assumed to be unity. Certainly the optical system is exposed to more than this limited FOV, but since the modulation of the spectrum only affects <u>amplitude</u>, the form of the obliquity function outside the FOV of interest can be ignored.

## Analysis of the New Constraint.

Gerchberg showed that when the finite extent of the object function is known, the superresolution algorithm does indeed work to continue the spectrum. He then tested the algorithm on the same object, but set the finite extent limits greater than the true limits. Although the spectrum was still continued somewhat, a definite modulation of the continued portion was observed such that the new spectrum approached zero with higher positive and negative frequencies. Because the new constraint developed, ie the semicircular space, is likely to be larger than any object function, this same behaviour is to be expected. Obviously this artifact will have a greater impact on actual spectra which have significant energy outside the band-limits of the optical system (ie high frequency components).

## Incoherent versus Coherent Illumination.

The above development applies to any object function which propagates as an infinite number of spherical waves. In the case of coherent illumination, the object function is the complex amplitude of the radiation leaving the object. The Fourier transform is therefore the direct superposition of those complex fields as they arrive at the optical system. In the case of incoherent illumination, the object function is the intensity of the radiation leaving the object. By the Van Cittert-Zernike Theorem, the Fourier transform of the normalized

object intensity distribution under incoherent illumination is the complex degree of spatial coherence (6:529). The problem of a band-limited spectrum is similar in both scenarios. The advantage of incoherent illumination is that the object function - the intensity - is always non-negative. This quality allows a further constraint for the superresolution algorithm.

## IV. SUPERRESOLVED IMAGING SIMULATIONS

The goal of superresolution is to develop a technique to improve resolution without increasing aperture size. Thus, the test of a superresolution technique shall be to determine if it can yield an image which has the resolution of a certain aperture, given the band-passed spectrum of a smaller aperture. Specifically, the band-passed spectrum from a 4 centimeter aperture shall be used to attempt to build a spectrum of a 32 centimeter effective aperture. The wavelength used is 1.5 mm. The incoherent scenario is applied due to its more abundant natural occurence.

Two one-dimensional test objects are used. The first is a two-point target with an offset center. The separation of the two points is chosen so that the points are unresolved in the image produced by the 4 cm aperture, but are certainly resolved with the 32 cm aperture. The second target is a wide, offset three-bar target somewhat like an Air Force resolution target. This type of object is more representative of typical extended objects. For each

object intensity distribution the Fourier transform is applied analytically, then the Hartley spectrum is generated using eqn (1) (representing the received complex degree of spatial coherence). The Hartley spectrum is then sampled over its center four centimeters at 64 equal spacings. These values are entered into the center of the initially zeroed 512 element RESULT array as the initial algorithm spectrum (512 elements are required for the 32 cm aperture). The additional constraint of a non-negative intensity is also applied in the algorithm.

The computer code used is described in Appendix B. The results are shown in Figures 3 and 4. Each plot in the left column is made up of the 512 values of the Hartley spectrum in frequency space. The plots in the right column illustrate the image constructed from the spectrum on its left. The top set illustrates the diffraction limited spectrum/image pair if an actual 32 cm aperture optical system is used. The center set illustrates the spectrum/image passed given by the 4 cm aperture. The spectrum of this set also represents the starting point of the algorithm. The bottom set is the result of the simulation after 1000 iterations.

The results of using the two-point test object are outstanding. It is apparent that with the q space limits wider than the object boundaries, the spectrum is modulated as predicted. The image of the three-bar test object is also remarkably improved, although not as dramatically as the two-point object results. Note the actual

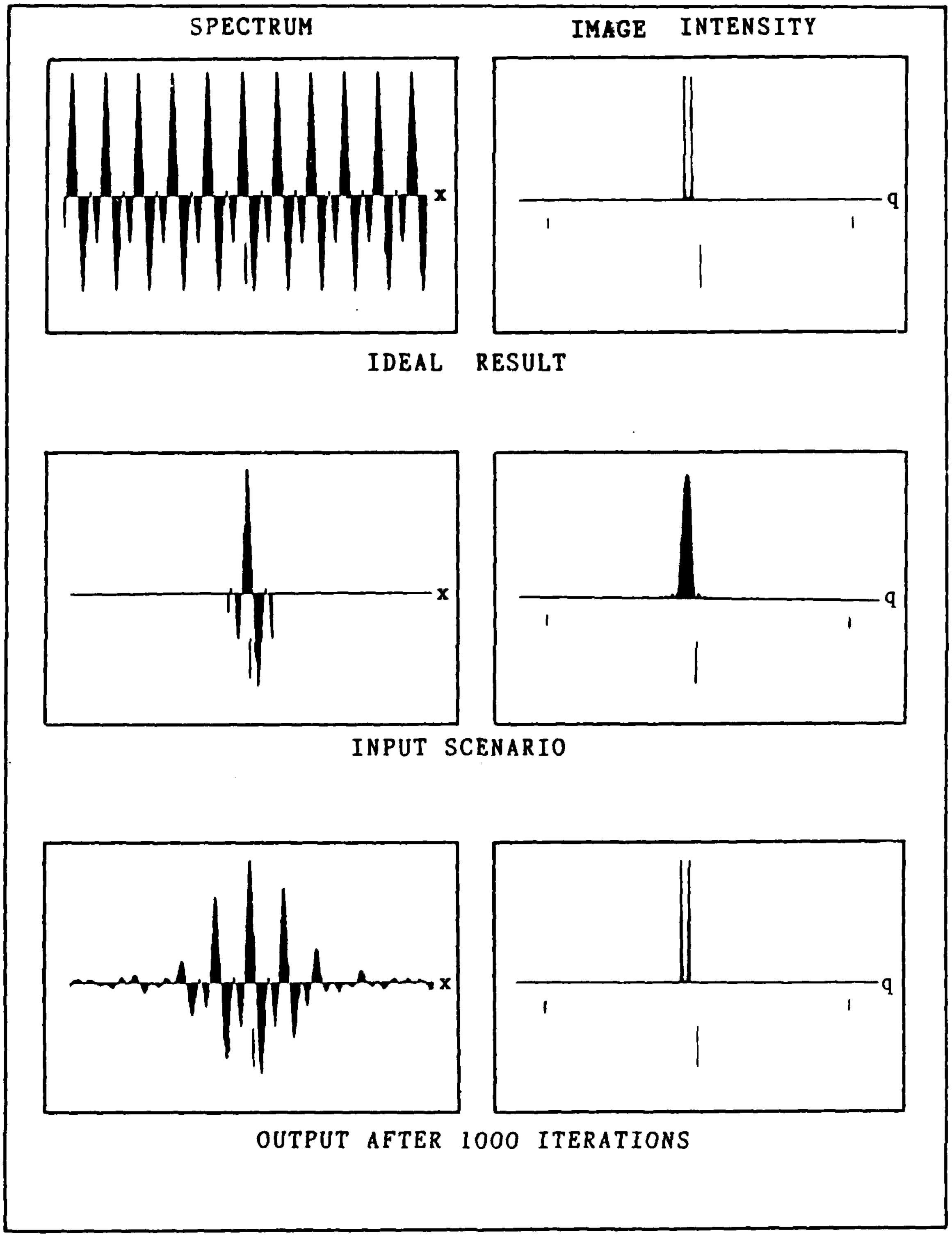


Figure 3. Two-point Object Superresolution Results.

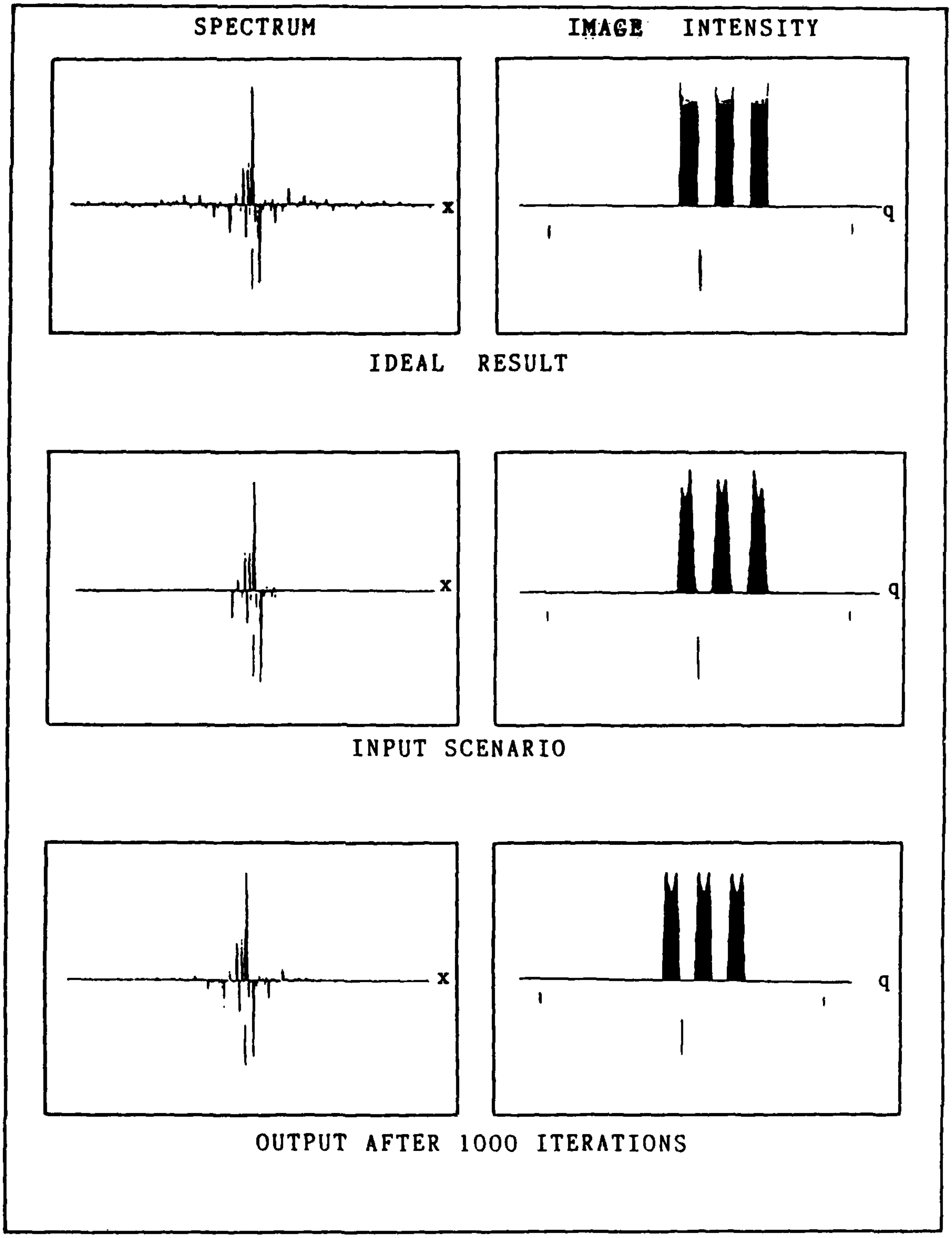


Figure 4. Three-bar Object Superresolution Results.

spectrum has most of its energy within the band-limited regime. Thus very little error energy is being subtracted upon each iteration.

The explanation for a modulation when the finite constraints are greater than the actual object function boundaries is hypothesized as follows. The growth of energy outside the band-limited spectrum can be considered to be the result of a transform of a finite, reconstructed image. The image is finite due to the truncation of energy outside the semicircular limits. Such a finite image must have a band-unlimited spectrum. Thus as the algorithm progresses, the energy "spilled" outside the band limitations is essentially due to the diffraction caused by truncating the image function. The truncation of error energy in q image space can be regarded as multiplying the generated image by a rect function the width of the constraint in that space. If the constraint locations match the actual object dimensions, then such a rect function would have no effect. If however, the rect function is larger due to a greater constraint such as the semicircular constraint, then its affect can not be ignored. The multiplication by the rect in q space is represented by a convolution of the image by a sinc function in frequency space. It is this convolution which allows energy to be spilled beyond the band-limited spectrum. Since the algorithm only truncates error energy in q space, the spectrum resulting from the spilled energy must be the emerging actual spectrum. In this view then, the sinc function, which is convolved with the band-limited spectrum, is responsible for determining how much energy is spilled. Note that the convolution is only

with the band-limited spectrum. Thus the maximum spillage will occur close to the band-limited spectrum, as illustrated in the results.

As the rect function representing the constraint in object space gets smaller, the convolving sinc function in frequency space will broaden, allowing more energy to spill outside the limits and increasing the growth of the continued spectrum. In the limit as the rect approaches the object's finite dimensions, the spillage will actually become the Fourier transform of the object and error functions only inside the object function domain. Thus as the constraints get closer to the actual constraints, the algorithm should converge quicker. Conversely, with the large rect function implied by a semicircular window, the algorithm should converge slowly since the spillage is more confined to near the band-limited spectrum. If this argument is true, it only implies a slowing of the algorithm, not necessarily a different final result. Theoretically, given enough time, the algorithm should converge to the same image regardless of how large the constraint window is (as long as it is larger than the actual window).

## V. CONCLUSIONS

The results indicate that superresolution is possible without a knowledge of the finite dimensions of the object. The requirement for the technique is a quasi-monochromatic, band-limited spectrum. This spectrum can be that generally associated with any valid transform relation between the object space and a frequency space which has

applicable constraints. The Hartley transform relation provides a working algorithm which is inherently faster and less memory intensive than the Fourier relation.

As a final note, the technique demonstrated is not necessarily confined to superresolution. Certain methods of phase retrieval are also modelled after the iterative approach. Since the examined constraint is in object space, any technique which benefits from object dimension constraints will benefit from this newly found constraint.

# APPENDIX A

## A BRIEF HISTORY OF SUPERRESOLUTION

This appendix is an account of the development of the field of superresolution. To be complete in itself, some of the development from the main thesis is duplicated.

It must be noted that many of the attempts for superresolution are concerned with the continuation of a known spectrum, even though actual data only yields the amplitude of the band-limited spectrum. The recovery of phase to construct the band-limited spectrum is a field in itself and will not be addressed in this background.

### The Classical Limit of Resolution.

Resolution traditionally refers to the ability to distinguish between closely spaced points in the object space. Thus a resolution limit is generally accepted as the minimum angular separation of two object points, as taken from the optical aperture, which yields an image which can be interpreted as that of two points. We therefore say that if the angular separation is less than this limit, the two points are unresolvable.

It was Sir George Airy (1801-1892) who first applied the Fraun-

hofer diffraction formula to a distant point source propagating through a circular aperture and discovered that the field distribution in the far field is a Bessel function - the intensity being later called an Airy pattern (6:419). In 1896, Lord John William Strutt Rayleigh generalized this interpretation to imaging.

> He envisaged each point on the object as a coherent source whose emitted wave was diffracted by the lens into an Airy pattern. Each of these in turn was centered on the ideal image point (on the image plane) of the corresponding point source. Thus the image plane was covered with a distribution of somewhat overlapping and interfering Airy patterns (6:563).

With this interpretation, Lord Rayleigh rather arbitrarily assigned the limiting resolution of a system as that angle between two point objects where the peak of one Airy pattern falls on the first minimum of the other.

In 1955, G. Toraldo Di Francia asserted that "...from the mathematical standpoint, the image of two points, however close to one another, is different from that of one point (7:497)." Upon this basis, Toraldo concluded that there is no theoretical limit of resolving power. Further, he claimed that the only true limit is a practical one which is related to the refinements of detector technology. From this point on virtually every reference to superresolution acknowledges this realization and Toraldo's achievement.

Approaches to Eliminate the Practical Limit.

Instead of reviewing the many efforts in superresolution in the past thirty years one by one, it is more efficient to categorize them along their general approaches and examine the combined contribution of each approach. The different categories include: information theory, analytic deconvolution, iterative error reduction and deconvolution by matrix methods.

Information Theory. In trying to create a specification for the practical limit of resolution, Toraldo examined the number of degrees of freedom which fully describe an object. The value of each of these degrees is the value of the spatial frequency distribution at points given by sampling theory. He concludes that since a true object spatial description must have an infinite number of degrees of freedom, and the image is in fact limited in spatial frequency due to the band-pass nature of the aperture, the complete object can never be recovered. He thus defines the limit of resolution as that resolution corresponding to the maximum degrees of freedom possible for the optical system. In fact, he claimed that there are many different objects which can have the same narrow band of observed spatial frequencies. He does admit however, that any knowledge about the object known a priori must yield more information. Thus this object knowledge is the only means of improving resolution beyond Toraldo's limit.

The basis of Toraldo's argument is that the optical system yields only a portion of the spatial frequency content of the object.

In 1963 J. L. Harris applied the mathematics of analytic functions to imaging (3). He showed that if an object has a definite size, then the analyticity of the observed spatial frequency function passed by the system can be continued throughout the frequency domain. Harris' conclusions were so important to the future of superresolution, that they are listed here directly from his paper.

> (1) For objects of finite angular dimensions, knowledge of the spatial frequency spectrum within the passband of any imaging system implies knowledge of the spatial frequency spectrum over the entire frequency domain, and hence implies complete knowledge of the object.
>
> (2) Two distinctly different objects of finite angular size cannot have identical images, so that the ambiguous image does not exist for realizable objects.
>
> (3) Diffraction, therefore, imposes a resolution limit which is determined by the noise of the system rather than by some absolute criterion.
>
> (4) Object detail, absent in a diffraction image, may be restored by a variety of techniques, with the ultimate limit in such restoration imposed by the system noise (3:963).

To complete the information theory approach to superresolution, W. Lukosz developed the techniques of exploiting these "limited" degrees of freedom (8;9). Lukosz discovered that certain information from the image can be enhanced at the expense of other information. Although these techniques are fascinating, the above arguments by Harris show that they are based on a false limit and therefore may be improved.

<u>Analytic Deconvolution.</u> In accordance with Lord Rayleigh's

description of an image, it can be shown that the intensity of an image point is simply the square of the convolution of the complex object distribution and the isoplanatic impulse response of the optical system (6:110). A technique to bring out the object function from the convolution integral and place it in terms of the image function is appropriately known as deconvolution.

In 1965, Casper W. Barnes, and in 1966, B. Roy Frieden derived an expression for the object distribution by deconvolution using orthogonal eigenequations for objects of limited angular width (10;11). Noting that the isoplanatic impulse response for a circular aperture is a Bessel function, they applied prior findings that the convolution integral in this case has a complete orthonormal set of prolate spheroidal eigenfunctions. Although the object function was brought out of the integral successfully, the resultant formula remained a short but involved expression of integrals and series. Frieden continued the treatment to the inclusion of noise. Although he showed favorable results for certain types of objects, he acknowledged simply that the limit of this type of reconstruction is noise. Similar conclusions to this effect were offered by C. K. Rushforth and R. W. Harris in 1967 (12). In this paper, dedicated to analyzing the noise of the analytical deconvolution method, it is shown that as the infinite series inside of the object integral is approximated with a larger upper limit to reduce truncation error, the noise inherent in the image becomes dominant, yielding a trade-off between resolution and noise. The instigator of the next method to be discussed inter-

preted these results as nothing less than condemning.

> Rushforth and Harris conclude that for this type of continuation [analytic deconvolution], the increase in resolution for a noisy diffraction limited spectrum, is offset by the amplification of the error in the reconstructed function to the extent that achieving resolutions much beyond the diffraction limit with realistically measured data, seems a very dubious proposition (1:711).

<u>Superresolution by Iterative Error Reduction.</u> In 1973, R. W. Gerchberg developed an attractive superresolution technique which spawned a relatively great deal of future work. An inverse Fourier transform of the band-limited spectrum of an object should reveal a diffraction limited image. Gerchberg shows that this diffraction blurs the boundaries of the image. If the boundaries of the object are known, then the error associated with the blurred image outside the known boundaries can be reduced. The procedure is best explained in his own words.

> The known portion of the spectrum is Fourier transformed to yield the diffraction limited [image] which is subsequently modified by setting all of the diffraction limited [image] outside the known extent of the true object to zero. Thus modified, the [image] is Fourier transformed and the generated spectrum is corrected so that that portion of the frequency range in which the true spectrum is known is given the correct values. The procedure iterates until a correction criterion based on the estimated [image] energy outside the known extent of the true object or the difference function energy between the estimated and true spectrum over the range of the known spectrum is satisfied (1:712).

Gerchberg continues to show that the error reduction per iteration decreases with continued iterations. He then considers

noise as a separate "spectrum" which can be handled independently due to the linearity of the algorithm and concludes that since noise is not analytic, the algorithm reduces the noise as well as the error energy (note: noise is not be considered in this thesis, although the Gerchberg algorithm is fundemental to the techniques pursued).

In 1983, John G. Walker applied the Gerchberg algorithm to coherent imaging and demonstrated its efficacy by experiment (13). Although the results yielded an image far better than diffraction limited, it was not in fact a true object representation. Walker hypothesized that discrete sampling, imperfect aperture edges, a finite sampling aperture, temporal instabilities and scattering were some of the reasons for the less than perfect result. Again, the only a priori knowledge was the finite extent of the object.

Deconvolution by Matrix Methods. A superposition integral can be converted into matrix equations exactly if the matrices describing the image and object are infinite and the impulse response of the system is known. While such infinite matrices are considered, this technique approaches analytical deconvolution (as a special case). Without considering the implications of infinite matrices, the superposition integral can be reformed into the simple equation $I = DO$, where I is the diffraction-limited image vector, O is the object vector and D is a matrix of values of the impulse response at a position defined by the matrix element positions of I and O. An interesting note is that these vectors can represent either intensity (incoherent

propogation) or complex functions (coherent propogation). Solving for the object vector yields

$$O = D^{-1}I \tag{A-1}$$

The problem resides in the fact that such an inversion problem is ill posed (14:149). Specifically, the calculation of $D^{-1}$ can not by found using traditional means due to truncation errors in the calculation and the presence of zeroes (or very small values) in the matrix to be inverted. A popular attempt to solve the problem without inversion is by minimizing the reformed equation $I - DO = 0$ . Such attemps were made by Crawford, et al, in 1981, and again by Stierwalt in 1985 using least squares methods (15;16). Again, although resolution was in fact enhanced, warnings were given that such a technique is not to be trusted in general, but that a knowledge of the object's extent will dramatically improve the results.

## Summary and Interpretation of Past Results.

The contributions of the four general approaches to super-resolution are fairly succinct. In the information theory approach, the contribution is the thought of trading off resolution of certain characteristics of the object for increased resolution of others. The analytical approach fails in the presence of noise. The iterative technique is successful only with the knowledge of object boundaries and the absence of radiation outside these boundaries. It also

presents an attractive insight on how to reduce error in the received diffracted distribution. The matrix method is very simple but has drawbacks due to the inversion problem of the diffraction matrix.

## APPENDIX B

## EXPLANATION OF COMPUTER CODE

The software language used was Turbo Pascal version 4.0. The computer used was an IBM compatible NEC multispeed laptop running at 9.54 MHz, without a math coprocessor. The source code is given at the end of the appendix. Comments are supplied here instead of in the listing to facilitate following the algorithm within the code. The time required for a 512 element FHT was just over 2 seconds.

### Definitions and Terminology

| | |
|---|---|
| W | The wavelength of incident radiation. |
| width | The physical dimension of the array representing the spectrum passed by the optical system. |
| numdet | The number of samples of the band-passed spectrum used in the simulation. |
| virtual samples | Those array elements which correspond to samples of frequency space outside the band-passed spectrum. |
| totaldet | The total number of array elements. |
| srlimitlo | The element in q space representing the left boundary (ie -W/2). |
| srlimithi | The element in q space representing the right boundary (ie W/2). |
| blstart, blfinish | The most negative and positive band-passed spectrum elements, repectively. |
| srstart, | The most negative and positive elements in the |

srfinish array used for superresolution, respectively.

<u>Description of Procedures.</u>

*Define_the_system.* This procedure is where the characteristics of the scenario are entered. For all experiments, the width shall be four cm; the wavelength, 1.5 mm; numdet, 64 elements; and totaldet, 512 elements. From these initial parameters, all results can be interpreted. The rather long wavelength is used to provide scenarios which can exhibit diffraction effects representative of the Rayleigh limit. Why this is necessary is obvious when one considers the resultant sampling in q space. Obviously we desire enough data points in the result to reasonably depict the image function. For example, a sinc function cannot be described reasonably by function values at only three points. The more points used to describe a function, the more the sampled function will resemble the actual function. Unfortunately with the FHT we cannot control the spacing of the samples in object space directly. The spacing between values in q space is equal to the inverse of the width between values in frequency space (another consequence of the discrete transform used). Since the size of q space where an object is allowed is limited to 2/W, it is obvious that only a certain number of equally spaced values are available to represent the reconstructed object. Another way of viewing the effect is that if the image has most of its energy between two samples, then it will be very difficult to interpret the success of the algorithm. Of course once a spectrum is achieved, a continuous Hartley transform can be performed to recover the object function at any

location in q space. This will not be done in this thesis.

*Initialize.* This procedure defines many of the parameters introduced in the above list of terms.

*Display.* This procedure simply outputs an axis, a center line, the limits in object space and any function desired onto a monitor display in 640 X 200 graphics mode.

*FHT.* This is the Fast Hartley Transform procedure. It is a modification of a source code presented by O'Neill (17).

*Three_Bar_Object* and *Two_Point_Object.* These procedures calculate the spectrum of the three-bar and two-point objects used in the experiments.

*Continue Spectrum.* This procedure represents the superresolution algorithm. The input to this procedure as shown is the band-limited spectrum calculated earlier.

Source Code

```
PROGRAM SUPERRESOLUTION;
USES crt,graph3;
  CONST      max_full_size = 513;        max_half_size = 257;
  TYPE
    type_1dc = array[-max_half_size..max_half_size] of real;
    type_2d = array[1..2,1..max_full_size] of real;
    direction_type = (direct,reverse); display_type = (object,spectrum);
  VAR
    result,htly_BL_spect : type_1dc;  perfunc : type_2d;
    per_mat : array[1..max_full_size] of integer;
    totaldet,srlimitlo,srlimithi,blstart,blfinish,srstart,srfinish,
```

```
    pwr,n2,ct,iteration,numdet : integer;
    incX,tmp,rt2,width,wavelength : real;
    converged,INITIALIZED : boolean;

PROCEDURE DEFINE_THE_SYSTEM;
BEGIN
  width         :=   4;                    {cm}
  numdet        :=   64;                   {must be 2**p}
  wavelength    :=   0.15;                 {cm}
  totaldet      :=   512;
END;

PROCEDURE INITIALIZE;
BEGIN
  rt2 := sqrt(2.0);
  srlimitlo := -(trunc(totaldet*width/(numdet*wavelength))+1);
  srlimithi := -srlimitlo;
  srstart := 1-(totaldet div 2);         srfinish := -srstart+1;
  blstart := 1-(numdet div 2);            blfinish := -prstart;
  for ct := -256 to 256 do
  begin
    htly_BL_spect[ct]:=0.0;  result[ct]:=0.0;
  end;
  incX := width/numdet;
END;

PROCEDURE DISPLAY(mat:type_1dc;typ:display_type);
VAR k:integer; inc,mx:real; ch:char;
BEGIN
  hires;clearscreen;draw(64,100,576,100,1); inc:=511/(totaldet-1);mx:=0;
  for k := 0 to totaldet-1 do
  begin
    if k=-srstart then draw(64+round(k*inc),130,64+round(k*inc),160,1);
    if typ=object then if ((k+srstart)=srlimitlo) or
       ((k+srstart)=srlimithi) then
         draw(64+round(k*inc),112,64+round(k*inc),120,1);
  end;
  if typ=object then for k := srstart to srfinish do
     mat[k]:=mat[k]*mat[k];
  for k:=srstart to srfinish do if abs(mat[k])>mx then mx:= abs(mat[k]);
  if mx>0 then for k := srstart to srfinish do mat[k]:=80*mat[k]/mx;
  if mx>0 then for k:= 0 to totaldet-1 do
    draw(64+round(k*inc),100,64+round(k*inc),100-
    round(mat[k+srstart]),1);
END;

PROCEDURE FHT(d_in:type_1dc; n:integer; direction:direction_type);
VAR
  dt : type_2d; ang,omega,mult : real;
  c,d,i,j,k,s,t,ta,fa,t_index,modify,s_start,s_end,power,trig_ind,
  trig_inc,temp_ind,i_temp,tp1,section : integer;
BEGIN
```

```
IF INITIALIZED=FALSE THEN
BEGIN
  INITIALIZED:=TRUE;
  pwr:=round(ln(n)/ln(2)); ang:=0.0; omega:=2.0*pi/n;  n2:= n div 2;
  for i := 1 to n do
  begin
    perfunc[1,i]:=sin(ang);  perfunc[2,i]:=cos(ang);  ang:=ang+omega;
    d:=0;   t_index:=i-1;
    for c := 1 to pwr do
    begin
      s := t_index div 2;      d := d+d+t_index-s-s;  t_index:=s;
    end;
    per_mat[i]:=d+1;
  end;
END;
power:=1; fa:=1; ta:=2;
for i:=1 to n2+1 do dt[fa,per_mat[i]]:=d_in[i-1];for i:=n2+2 to n do
  dt[fa,per_mat[i]]:=d_in[i-n-1];
for i := 1 to pwr do
begin
  j:=1; section:=1; trig_inc := n div (power+power);
  while j <= n do
  begin
    trig_ind:=1; s_start:=section*power+1; s_end:=(section+1)*power;
    tp1:=s_start+s_end+1;
    for k := 1 to power do
    begin
      t:=j+power; if (s_start=t) or (pwr<3) then modify:=t else
                                                  modify:=tp1-t;
      dt[ta,j]:=dt[fa,j]+dt[fa,t]*perfunc[2,trig_ind]+
      dt[fa,modify]*perfunc[1,trig_ind];
      temp_ind:=trig_ind+n2;
      dt[ta,t]:=dt[fa,j]+dt[fa,t]*perfunc[2,temp_ind]+
      dt[fa,modify]*perfunc[1,temp_ind];
      trig_ind:=trig_ind+trig_inc;    j:=j+1;
    end;
    j :=j+power;   section:=section+2;
  end;
  power:=power+power; i_temp:=ta; ta:=fa; fa:=i_temp;
end;
if direction=direct then mult:=1.0*n else mult:=1.0;
for i := 1 to 1+n2 do result[i-1]:=dt[fa,i]/mult;
for i := n2+2 to n do result[i-n-1]:=dt[fa,i]/mult;
END;

PROCEDURE TWO_POINT_OBJECT; var a1,a2 : real;
BEGIN
  a1:=0.050; a2:=0.2;
  for ct := prstart to prfinish do
  begin
    htly_BL_spect[ct]:=cos(2*pi*a1*ct*incX/wavelength)+
      cos(2*pi*a2*ct*incX/wavelength)-sin(2*pi*a1*ct*incX/wavelength)-
```

```
      sin(2*pi*a2*ct*incX/wavelength);
  end;
END;
PROCEDURE THREE_BAR_OBJECT; var A,B,C : real; amp,phase : type_1dc;
BEGIN
  A :=0.12; C:=0.15; B:=0.23;
  for ct := prstart to prfinish do
  begin
    if ct<>0 then
    begin
      amp[ct]:=(sin(pi*A*ct*incX/wavelength)/(pi*A*ct*incX/wavelength))*
               (1+2*cos(2*pi*ct*incX*B/wavelength));
    end else amp[ct]:=3.0;
    phase[ct]:=cos(2*pi*ct*incX*C/wavelength)+
               sin(2*pi*ct*incX*C/wavelength);
    htly_BL_spect[ct]:=amp[ct]*phase[ct];
  end;
END;

PROCEDURE CONTINUE_SPECTRUM;
BEGIN
  INITIALIZED:=FALSE;
  iteration := 0;     converged:=false;
  while converged=false do
  begin
    for ct := prstart to prfinish do result[ct] := htly_BL_spect[ct];
    FHT(result,totaldet,reverse);
    for ct := srstart to srlimitlo do result[ct]:=0.0;
    for ct := srlimithi to srfinish do result[ct]:=0.0;
    for ct := srlimitlo to srlimithi do if result[ct]<0.0 then
      result[ct]:=0.0;
    FHT(result,totaldet,direct); gotoxy(1,1);writeln(iteration);
    iteration:=iteration+1;
    if iteration=1000 then converged:=true;
  end;
END;

BEGIN                                         { MAIN PROGRAM SECTION }
  define_the_system;
  initialize;
  {two_point_object; OR} three_bar_object;
  continue_spectrum;
END.
```

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Resolution in an image can be increased by an iterative technique (introduced by Gerchberg) which effectively continues the known, partial spectrum beyond the limits imposed by an optical system. This increased resolution is called superresolution. Historically, an important constraint assumed for this technique was the knowledge of the finite dimensions of the object such that the object energy outside these dimensions must be zero. It is shown in this thesis that by a change of object space geometry, the semicircular field of view of an optical system provides a natural dimensional constraint which can be used instead of the object dimensions to achieve superresolution. A further modification of the iterative technique involves using the Fast Hartley Transform (FHT) instead of the Fast Fourier Transform (FFT). The FHT is inherently faster and requires less computer memory than the FFT.